MATH 1200: Review for Final

1. Prove that for any positive integer number n , $n^3 + 2n$ is divisible by 3.

2. Show that if $a_n = 2a_{n-1} + (-1)^n$ and $a_0 = 2$ then show that $a_n = (5 \cdot 2^n + (-1)^n)/3$.

3. Define $R = \{(x, y) : x, y \in \mathbb{C}, \exists a \in \mathbb{R}, a \neq 0, ax = y\}$. The set R is know as a relation on $\mathbb{C}$.

   (a) Show that $(x, x) \in R$ for all $x \in \mathbb{C}$. (that is, show that R is reflexive).

   (b) Show that for any $x, y \in \mathbb{C}$, if $(x, y) \in R$, then $(y, x) \in R$ (that is, show that R is symmetric).

   (c) Show that for any $x, y, z \in \mathbb{C}$, if (x, y) and $(y, z) \in R$, then $(x, z) \in R$ (that is, show that R is transitive).

   (d) Let $C_x = \{y : (x, y) \in R\}$. On a graph of the complex plane, draw all of the points in C_0. On a separate graph of the complex plane draw all of the points in C_1 and all of the points in C_{1+i}.

4. Find integers r and s such that $309r + 1234s = 1$.

5. Find all solutions to the equation $309x \equiv 5 \pmod{1234}$.

6. Show that for $n \geq 1$,

$$\frac{1}{1 \cdot 5} + \frac{1}{5 \cdot 9} + \cdots + \frac{1}{(4n-3)(4n+1)} = \frac{n}{4n+1} \ .$$

7. For all integers $n \geq 2$, $\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right)\cdots\left(1 - \frac{1}{n^2}\right) = \frac{n+1}{2n}$.

8. Prove or disprove the following statements:

   (a) Let a and n be integers, if $a|n$, then $a|n^2$.

   (b) Let a and n be integers, if $a|n^2$, then $a|n$.

   (c) Let a, n, m be integers, if $a|n$ or $a|m$, then $a|(n \cdot m)$,.

   (d) Let a, n, m be integers, if $a|(n \cdot m)$, then $a|n$ or $a|m$.

   (e) Let a, n, m be integers and $gcd(n, m) = 1$, if $a|(n \cdot m)$, then $a|n$ or $a|m$.

   (f) Let a, n, m be integers and $gcd(a, n) = 1$, if $a|(n \cdot m)$, then $a|n$ or $a|m$.

9. Find all $z \in \mathbb{C}$ such that $z^2 = i$.

10. Let x, y be real numbers and a, b, c be integers. For the following statements, either prove that they are true or provide a counterexample:

    (a) If $x + y$ is irrational, then at least one of x or y is irrational.

(b) If $x + y$ is rational, then both x and y are rational.

(c) Between any two rational numbers there is a rational number.

(d) For all real numbers x, there is a y such that $x \cdot y$ is rational.

11. For which values of n is $(1 - i)^n$ real? What values of n make it imaginary?